647510 **BSCmaMJ203x-2C** **Seat No :_____**

B.Sc. Semester - 2 (NEP-2020) Examination

MARCH/APRIL-2025

MATH: Theory of Ordinary Differential Equations (Major-3)

Time: 2:00 Hours **Marks: 50**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Que-1(A) Answer any one: (05)

(1) Derive solution of Bernoulli's differential equation $\frac{dx}{dy} + Px = Qx^n$, where P & Q are functions of y.

(2) Solve: $xy(1 + xy^2)\frac{dy}{dx} = 1$

Que-1(B) Answer any one: (05)

(1) State and prove the necessary condition for a differential equation $Mdx + Ndy = 0$ to be an exact differential equation (where M & N are functions of x & y).
Also solve:$(sinx\ siny + sec^2x)dx - (cosx\ cosy - tan^2y)dy = 0$.

(2) Define: Orthogonal Trajectories.
Find orthogonal trajectories to circles centered at the origin if possible.

Oue-2(A) Answer any one: (05)

(1) Write the general form of ordinary differential equations of first order and higher degree and discuss about the solution of the differential equations which are solvable for p, where $p = \frac{dy}{dx}$.

(2) Solve: $x^2 = a^2(1 + p^2)$.

Que-2(B) Answer any one: (05)

(1) State Clairaut's and Lagrange's forms of ordinary differential equations.
Describe the process of solving Lagrange's differential equations.
Explain: Clairaut's differential equation is a particular case of Lagrange's differential equation.

(2) Solve: $y^2 = xyp + \left(\frac{yp}{x}\right)^3$.

Que-3(A) Answer any one: (05)

(1) Write the general form of the linear differential equation of n^{th} order.
Explain the process of finding its solution.

(2) Prove: $\frac{1}{f(D)}e^{ax}V = e^{ax}\frac{1}{f(D+a)}V$, where V is a function of x.

Que-3(B) Answer any one: (05)

(1) Solve: $[(D + 1)(D - 3)^2]y = e^{3x} + e^{5x}$.

(2) Solve: $(D^2 - 2D + 1)y = xe^x sinx$.

Que-4(A) Answer any one: (05)

(1) Give general form of homogeneous linear differential equation with variable coefficients.
Explain a method of finding its Complementary Function (CF).

(2) Solve: $(\boldsymbol{x^3D^3 + 3x^2D^2 - 6xD + 6})\boldsymbol{y} = \left(\boldsymbol{x^2 - \frac{1}{x^2}}\right)^{\boldsymbol{2}}$.

Que-4(B) Answer any one: (05)

(1) If m is the r times repeated root of $f(\theta)$, then prove that $\frac{\boldsymbol{1}}{\boldsymbol{f(\theta)}}\boldsymbol{x^m} = \frac{\boldsymbol{x^m}}{\boldsymbol{r!}}\frac{\boldsymbol{(logx)^r}}{\boldsymbol{F(\theta)}}$; $\boldsymbol{where\ f(\theta) = (\theta - m)^r F(\theta)}$.

(2) Solve: $(\boldsymbol{x^2D^2 + 2xD - 20})\boldsymbol{y} = \boldsymbol{(x+1)^2}$

Que-5(A) Answer any one: (05)

(1) Define: Singular Solution of an ordinary differential equation. Explain any of the methods to find singular solution by means of an example.

(2) Solve: $\frac{\boldsymbol{d^4y}}{\boldsymbol{dx^4}} + \boldsymbol{4y} = \boldsymbol{0}$.

Que-5(B) Answer any one: (05)

(1) Define: Complementary Function (CF) and Particular Integral (PI).

(2) Solve: $(\boldsymbol{x^2D^2 + 1})\boldsymbol{y} = \boldsymbol{5x^3}$.

****